Downlink RIS-NOMA with Constellation Adjustment for 6G Wireless Communication Systems

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***Abstract*—We consider a downlink reconfigurable intelligent surfaces-based non-orthogonal multiple access technique with constellation adjustment (RIS-NOMA-CA) for 6G wireless communication systems. We mathematically analyze the bit-error-rate (BER) performance of the RIS-NOMA-CA technique assuming the optimal joint maximum likelihood (JML) detector at receivers. To the best of our knowledge, theoretical analysis of the downlink RIS-NOMA-CA technique has not been reported in the literature. We show that the RIS-NOMA-CA technique yields a better BER performance than the conventional RIS-NOMA technique without CA through computer simulations, and our analytical result matches well with simulation results.**

***Index Terms*—Reconfigurable intelligent surface (RIS), non–orthogonal multiple access (NOMA), bit-error-rates (BER).**

I. Introduction

For 6G wireless communication systems, a reconfigurable intelligent surface (RIS) has emerged as a promising technology to cope with a severe propagation-loss due to obstacles between transmitter and receiver in high-frequency bands [1]. Several studies on application of the RIS to non-orthogonal multiple access (NOMA) technique have been conducted to improve quality of services (QoS) including spectral efficiency, fairness, connection capacity, energy efficiency, etc [2], [3]. In particular, a constellation adjustment (CA) technique was applied to *uplink* RIS-NOMA for improving the error performance in [3], but the CA has not been applied to downlink RIS-NOMA technique. Furthermore, a bit-error-rate (BER) performance of RIS-NOMA with CA has not been theoretically analyzed in both uplink and downlink.

In this paper, we consider a downlink RIS-NOMA system with CA and mathematically analyze the BER performance of the downlink RIS-NOMA with CA. We show that the RIS-NOMA with CA significantly outperforms the conventional RIS-NOMA without CA in terms of BER through extensive computer simulations.

II. Downlink RIS-NOMA with CA

Fig. 1 illustrates a downlink RIS-NOMA systems consisting of a single base station (BS) with a single antenna, M user equipments (UEs) with a single antenna, and a single RIS with N elements. We assume that N elements of the RIS are partitioned into M sub-surfaces and the $i \in \{1, \cdots, M\}$-th subsurface with N_i elements supports the i-th UE. The term $\mathbf{h}_i (\in \mathbb{C}^{N_i})$ represents the wireless channel vector from the i-th subsurface to the BS. We define $\mathbf{g}_{m,i} (\in \mathbb{C}^{1\times N_i})$ as the wireless channel vector from the i-th subsurface to the $m \in \{1, \cdots, M\}$-th UE. We assume that $\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, l_{\mathrm{BR}}^{\alpha}\mathbf{I}_{N_i})$ and $\mathbf{g}_{m,i} \sim \mathcal{CN}(\mathbf{0}, l_m^{\alpha}\mathbf{I}_{N_i})$ where l_{BR}, l_m and α denote the distance from the BS to the RIS, the distance from the m-th UE to the RIS, and the path-loss exponent, respectively. It is assumed that there exists no direct link between the BS and each UE. Let the $n(\in \{1, \cdots, N_i\})$-th element of $\mathbf{h}_i$ and $\mathbf{g}_{m,i}$ be $h_{i,n}$ and $g_{m,i,n}$, respectively. The phase of each element is defined as $\varphi_{i,n} := \angle h_{i,n}$ and $\phi_{m,i,n} := \angle g_{m,i,n}$. The phase shift coefficients of the i-th subsurface in the RIS can be represented with a diagonal matrix $\mathbf{\Theta}_i (\in \mathbb{C}^{N_i \times N_i})$ to maximize the received signal-to-noise ratio (SNR) at the i-th UE, where the n-th diagonal component of $\mathbf{\Theta}_i$ is set to $e^{-j(\varphi_{i,n}+\phi_{i,i,n})}$. Then, the received signal at the m-th UE is given by

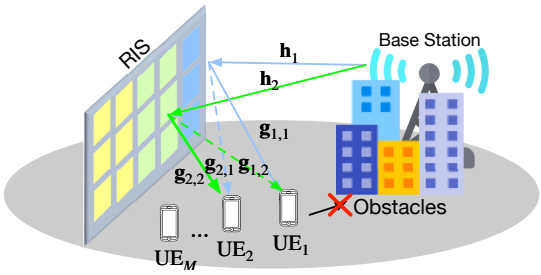

Fig. 1. System model of downlink RIS-NOMA with CA including a BS, M UEs, and an RIS with N elements.

$$y_m = \sum_{i=1}^{M} \mathbf{g}_{m,i}\mathbf{\Theta}_i\mathbf{h}_i x + w_m, \tag{2}$$

where w_n denotes the additive noise, i.e., $w_m \sim \mathcal{CN}(0, \sigma^2)$ and $x \in \mathbb{C}$ denotes the transmit signal of the BS. We consider the CA technique with QPSK modulation to improve the BER performance of the RIS-NOMA system. The term x represents the magnitude- and phase-adjusted weighted sum of QPSK symbols transmitted over NOMA, i.e., $\sum_{m=1}^{M}\sqrt{\tilde{a}_m P}s_m e^{j\delta_m}$, where δ_m, $\tilde{a}_m$, s_m and P represent angle of the steering phase, power allocation factor for m-th UE, normalized QPSK symbol for m-th UE, and transmit power, respectively [4]. The m-th UE tries to detect symbols $\hat{\mathbf{s}} = [\hat{s}_1, \cdots, \hat{s}_M]$ by utilizing the joint maximum likelihood (JML) detector as

$$\hat{\mathbf{s}} = \arg\min_{s_1,\cdots,s_M \in \chi} \left| y_m - \sum_{i=1}^{M} \mathbf{g}_{m,i}\mathbf{\Theta}_i\mathbf{h}_i x \right|^2, \tag{3}$$

where χ is a candidate set of normalized QPSK symbols, i.e., $\chi \triangleq \{c_1, c_2, c_3, c_4\} = \{\frac{-1+j}{\sqrt{2}}, \frac{-1-j}{\sqrt{2}}, \frac{1+j}{\sqrt{2}}, \frac{1-j}{\sqrt{2}}\}$ [5].

III. Error Performance Analysis

For the mathematical simplicity, we assume that the BS serves two UEs through the RIS in this section. Then, we denote a superimposed QPSK symbol as $C_k = e^{j\delta_1}\sqrt{\tilde{a}_1}c_{\lceil k/4 \rceil} +$

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$$\Pr(\varepsilon_1) \le \frac{1}{4}\sum_{p=1}^{4}\sum_{q=5}^{16}\mathbb{E}_{\rho_1}\left[e^{-t_1\left(\rho_1\sqrt{\frac{D_{p,q}^2\gamma}{2}}\right)^2 - t_2\left(\rho_1\sqrt{\frac{D_{p,q}^2\gamma}{2}}\right) - t_3}\right], \Pr(\varepsilon_2) \le \frac{1}{4}\sum_{p=1}^{4}\sum_{q\notin\mathcal{Q}_p}\mathbb{E}_{\rho_2}\left[e^{-t_1\left(\rho_2\sqrt{\frac{D_{p,q}^2\gamma}{2}}\right)^2 - t_2\left(\rho_2\sqrt{\frac{D_{p,q}^2\gamma}{2}}\right) - t_3}\right], \quad (1)$$

$$\mathbb{E}_{\rho_m}\left[e^{-t_1\left(\rho_m\sqrt{\frac{D_{p,q}^2\gamma}{2}}\right)^2 - t_2\left(\rho_m\sqrt{\frac{D_{p,q}^2\gamma}{2}}\right) - t_3}\right] = \frac{e^{-\frac{2t_3v_m+\mu_m^2}{2v_m}}\cdot e^{\frac{(t_2v_mD_{p,q}\sqrt{\gamma}-\sqrt{2}\mu_m)^2}{4t_1v_m^2D_{p,q}^2\gamma+4v_m}}}{2\sqrt{t_1v_mD_{p,q}^2\gamma+1}}\left[1+\mathrm{erf}\left(\sqrt{\frac{(t_2v_mD_{p,q}\sqrt{\gamma}-\sqrt{2}\mu_m)^2}{4t_1v_m^2D_{p,q}^2\gamma+4v_m}}\right)\right].$$

$e^{j\delta_2}\sqrt{\tilde{a}_2}c_{((k-1)\bmod 4)+1}$, $k \in \{1, 2, \cdots, 16\}$ [4]. We exploit the optimal phase difference, i.e., $\delta_1 - \delta_2$, and power allocation factor $\tilde{a}$ which were derived in [3]. Assuming that the RIS has a large number of elements, an effective channel of the m-th UE, $\psi_m = \sum_{i=1}^{2}\mathbf{g}_{m,i}\mathbf{\Theta}_i\mathbf{h}_i$, can be statistically modeled as the sum of non-central normal Gaussian and complex Gaussian distribution by central limit theorem [2]. Then, since phase shift coefficients of the RIS are designed to align phases of all wireless channels corresponding to each UE to maximize the received SNR, the real value of the effective channels containing the desired channel aligned by the RIS might be much larger that the imaginary value. Hence, an upper bound for the BER can be simply derived by reducing the channel amplitude of the effective channel $|\psi_m|$ to the real part $\Re\{\psi_m\}$. For the ease of explanation, let $\rho_m := \Re\{\psi_m\}$. Then, ρ_m follows normal distribution where a mean of μ_m and variance of v_m are derived as

$$\mu_m = N_m\sqrt{l_{\mathrm{BR}}^{\alpha}l_m^{\alpha}}\frac{\pi}{4}, v_m = l_{\mathrm{BR}}^{\alpha}l_m^{\alpha}\left\{N_m\left(1-\frac{\pi}{16}\right)+\frac{(N-N_m)}{2}\right\}. \quad (4)$$

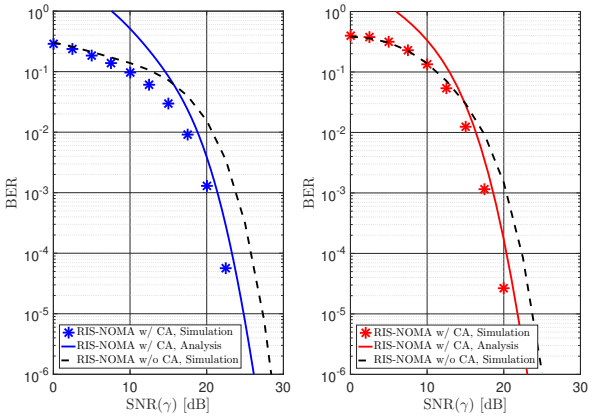
With the symmetry of QPSK constellations, we derive the error probability of the first UE when transmitting symbol c_1 without loss of generality. Then, the union bound of the conditional BER given ρ_1, can be expressed as

$$\Pr(\varepsilon_1|\rho_1) \le \frac{1}{4}\sum_{p=1}^{4}\sum_{q=5}^{16}Q\left(\rho_1\sqrt{\frac{D_{p,q}^2\gamma}{2}}\right), \quad (5)$$

where $D_{p,q}$ denotes the Euclidean distance between superimposed constellation point C_p and C_q where $(p, q \in \{1, 2, \cdots, 16\})$ and $\gamma := P/\sigma^2$. Since (5) is mathematically intractable in deriving a closed form of the BER performance, we exploit a substitute of the Q-function which is known to be a tight approximation as $Q(x) \approx e^{-t_1x^2-t_2x-t_3}$ in [6] where t_1, t_2, and t_3 denote fitting parameters. Consequently, the upper bound of the BER of the RIS-NOMA with CA in the case of two UEs can be derived as (1) where $\mathcal{Q}_p$ denotes an index set of constellations that do not cause errors from the perspective of the second UE when the BS transmits C_p.

IV. Simulation Results

In this section, we validate the BER performance of downlink RIS-NOMA with CA through Monte-Carlo simulations and we verify analytical result on the BER of the presented system by comparing with the simulation result. For simulation setup, We set l_{BR} to 20m, l_1 to 6m, l_2 to 4m, the number of elements N in the RIS to 200, and the path loss exponent to 2. We consider an environment where two UEs exist, and the number of elements of a subsurface within the RIS to serve each UE is 100, i.e., $N_1 = N_2 = 100$. Fig. 2 shows the



(a) UE 1 (b) UE 2

Fig. 2. BER performance of downlink RIS-NOMA with CA when $N_1 = N_2 = 100$ and $(l_{\mathrm{BR}}, l_1, l_2) = (20\mathrm{m}, 6\mathrm{m}, 4\mathrm{m})$.

BER performance of downlink RIS-NOMA with CA technique for varying SNR at (a) UE 1 and (b) UE 2, respectively. It can be seen that applying CA can achieve significant BER performance improvement compared to the case where CA is not applied in the downlink RIS-NOMA system. In addition, it is worth noting that our mathematical analysis is matched well with the simulation result.

V. Conclusion

We presented a downlink RIS-NOMA with constellation adjustment technique including signal rotation and scaling. We also mathematically analyzed the BER performance of the presented system by utilizing the characteristics of RIS with a large number of elements. Through computer simulations, we confirmed that a significant BER performance improvement can be achieved via CA in downlink RIS-NOMA systems.

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